

The effects of the Moon's shadow and pre-shadow on the artificial satellites motion

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Abstract

The perturbations due to the solar radiation pressure forces on the Earth's artificial satellites have been given taking into account the Moon's shadow and pre-shadow effects. The formulae are convenient when the perturbed forces are obtained by numerical methods. The study shows that the cylindrical model of the shadow is insufficient and that the conic shadow model including the pre-shadow gives more accurate results.

Introduction

The artificial satellites, in their motion around the Earth, can sometimes enter the Moon's shadow.^{1,2} The probability of the satellite's entry the Moon's shadow can not be considered small,^{3,4} as it is equal to 5% for the near-equatorial orbits with semi-major axes of 10-15 radii of the Earth. The problem of pre-computing the Luni-Solar eclipses for the geostationary satellites was considered by several authors.⁵⁻⁸ The estimation of the influence of the solar eclipses on the Earth's satellites has been obtained by some previous works.⁹⁻¹¹

The influence of the Moon's shadow and pre-shadow effects on the motion of the Earth's satellite will be studied and the corresponding estimation will be given.

Theoretical model

The perturbing acceleration

Let us suppose that the satellite is homogeneous and has a spherical surface and a constant mass. Then, the perturbing acceleration of such satellite due to solar light pressure can be given as:¹²

$$\vec{F}^* = k \left(\frac{A}{\Delta_s} \right)^2 C_R \left(\frac{s}{m} \right) \left(\frac{\vec{\Delta}_s}{\Delta_s} \right) \quad (1)$$

where k is the solar constant and $k(\approx 4.5605 \times 10^{-6} \text{ Newton/meter}^2)$;

A is the semi-major axis of the Earth's orbit;

$\vec{\Delta}_s$ is the vector of the heliocentric position of the satellite;

$\Delta_s = |\vec{\Delta}_s|$, s is the area of the largest section of the satellite (the middle area);

m is the satellite's mass;

C_R is a constant, which value lies between 0 and 2, depending on the reflection properties of the satellite's surface.

Let

$$k \approx 4.5605 \times 10^{-6}, C_R = 1.44, A = 1.496 \times 10^{11} \text{ m}, \left(\frac{s}{m} \right) = 6.9 \times 10^{-4} \text{ m}^2/\text{kg}$$

So, substituting these values into equation (1), taken into account the various values of Δ_s , the corresponding \vec{F}^* is obtained

$$\Delta_s = 1.227 \times 10^7 \text{ m}, \text{ the resultant } \vec{F}^* \approx 0.067 \text{ m}$$

$$\Delta_s = 1 \times 10^7 \text{ m}, \text{ the resultant } \vec{F}^* \approx 1.014 \text{ m and}$$

$$\Delta_s = 0.8 \times 10^7 \text{ m}, \text{ the resultant } \vec{F}^* \approx 1.58 \text{ m.}$$

It must be mentioned that equation (1) corresponds to the case when the satellite is lightened by the non-eclipsed Sun. If the solar disk is partly or fully eclipsed by the Moon or by the Earth, then, the perturbing acceleration due to the solar-light pressure may be written as:

$$\vec{F}^* = f \vec{F}^* \quad (2)$$

Where the function f is the so-called shadow function,¹³ and \vec{F}^* is given by the equation (1). The value of f is within the limits of 0 and 1 and depends on both the shadow functions of the Earth f_E and of the Moon f_L . Here we give formula for f_L only.

Conditions for satellite to be in the Moon's shadow and semi-shadow

The Moon and the Sun are supposed to have spherical surfaces with radii R_L and R_s , respectively. Let $\vec{\Delta}_L$ be the seleno-centric vector of the satellite position and also let us consider that $\Delta_L = |\vec{\Delta}_L|$. The angle between the directions from the satellite to the Sun and to the Moon is ψ . Therefore, ψ is determined by the formula:

$$\sin \psi = \frac{|\vec{\Delta}_s \times \vec{\Delta}_L|}{\Delta_s \Delta_L}; \quad \cos \psi = \frac{\vec{\Delta}_s \cdot \vec{\Delta}_L}{\Delta_s \Delta_L}, \quad 0 \leq \psi \leq \pi \quad (3)$$

The equations of the lightening of the satellite are determined by the mutual positions of the disks of the Sun and the Moon as seen from the satellite. Let ω_s and ω_L be the angular radii of the Sun and the Moon respectively, as seen from the satellite, then we have:

$$\sin \omega_s = \frac{R_s}{\Delta_s}; \quad \sin \omega_L = \frac{R_L}{\Delta_L} \quad (4)$$

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Accordingly, three kinds of mutual positions can be distinguished:

i) The satellite is out of the shadow and pre-shadow of the Moon (Figure 1), it means that the Sun's disk as seen from the satellite is not covered by the Moon.

In this case we have:

$$\psi \geq \omega_L + \omega_s \quad (5)$$

ii) The partial solar eclipse is seen from the satellite. In this case the Moon's disk as seen from the satellite covers only a part of the solar one and we can write the following condition:

$$\omega_L + \omega_s > \psi > |\omega_L - \omega_s| \quad (6)$$

iii) The full or the ring eclipse of the Sun is seen from the satellite. In this case, we have:

$$\psi \leq |\omega_L - \omega_s| \quad (7)$$

Figure 1 illustrates all the three cases.

Let us suppose that the intensity of the solar radiation is constant over the whole solar disk. Then, according to the work of Kabelac,¹⁴ we can write f_L in the form

$$f_L = \frac{\sigma}{\sigma^*} \quad (8)$$

Where σ^* is the area of the solar disk when it is not covered by the Moon as seen from the satellite and σ is the area of the non-eclipsed part of the solar disk as seen from the satellite. When the satellite is out of the shadow and pre-shadow of the Moon, then we have

$$\sigma = \sigma^* \Rightarrow f_L = 1 \quad (9)$$

The most complex formula for f_L takes place when the partial solar eclipse is seen from the satellite (the second case). In this case the angles ω_L and ω_s are small (<0.01 radius), so with error less than 1% the part of the unit sphere can be represented as a plane. The corresponding formula is:

$$\sigma^* = \pi \omega_s^2 \tag{10}$$

Considering now that the Moon begins to cover the Sun, as seen in Figure 2, therefore, the angular radius ω_s is continuously decreasing to zero value. Also the distance between the two centers of the Moon and the satellite is decreasing.

From the figure, we have the following relations describing these behaviors:

$$\omega_s + \omega_L = Z \tag{11}$$

and

$$(\omega_s + \omega_L)^2 = Z^2 = X^2 + Y^2 - 2XY \cos \psi \tag{12}$$

So, we have

$$\cos \psi = \frac{Z^2 - X^2 - Y^2}{-2XY}$$

When σ^* and σ are known, then f_L can be determined using equation (8). Finally, the third case, when the full eclipse of the Sun by the Moon is seen from the satellite, we have

$$f_L = \begin{cases} 0 & ; \omega_s \leq \omega_L \\ 1 - \left(\frac{\omega_L}{\omega_s}\right)^2 & ; \omega_s > \omega_L \end{cases} \tag{13}$$

During the solar eclipse the satellite is undergoing the radiation pressure in the direction from the effective center of the non-eclipse part of the solar disk to the satellite. Deflection of this direction from the center of the Sun to the satellite line is not more than the visible Sun's angular radius (<0.005 radians). The effect of this deflection in the perturbing acceleration of the satellite is no more than 1% of the direct solar pressure. This effect can be neglected with no harm to the accuracy of calculations.

Numerical estimation of the model

The model example has been taken in which the satellite moves around the Earth in an equatorial orbit with the semi-major axis $A=1.227 \times 10^7$ meter (Figure 1).

For the parameters in equations (1) and (4), the following values have been taken:

$$C_R = 1.44$$

$$\frac{s}{m} = 6.9 \times 10^{-4} \text{ m}^2 / \text{kg}$$

$$R_L = 1.738 \times 10^6 \text{ m}$$

$$R_s = 6.96 \times 10^8 \text{ m}$$

It is supposed that the Sun and the Moon move in the plane of the satellite's orbit along circular orbits, with the following radii:

$$a_s = 1.496 \times 10^{11} \text{ m};$$

$$a_L = 3.840 \times 10^8 \text{ m}$$

with the angular velocity:

$$V_s = 0.0172 \text{ radian / day};$$

$$V_L = 0.23 \text{ radian / day}$$

Three variants of the initial positions of the Sun (x_s^0, y_s^0) and of the Moon (x_L^0, y_L^0) have been taken as follows:

i) $x_s^0 = 1.496 \times 10^{11} \text{ m}, y_s^0 = 6.4 \times 10^8 \text{ m}, x_L^0 = 3.843 \times 10^8 \text{ m},$

$$y_L^0 = 8.5 \times 10^6 \text{ m}$$

ii) $x_s^0 = 1.496 \times 10^{11} \text{ m}, y_s^0 = 6.4 \times 10^8 \text{ m}, x_L^0 = 3.843 \times 10^8 \text{ m},$

$$y_L^0 = 6.7 \times 10^6 \text{ m}$$

iii) $x_s^0 = 1.496 \times 10^{11} \text{ m}, y_s^0 = 6.4 \times 10^8 \text{ m}, x_L^0 = 3.843 \times 10^8 \text{ m},$

$$y_L^0 = 3.8 \times 10^6 \text{ m}$$

The satellite's initial coordinates have been taken to be:

$$x^0 = 0, y^0 = -1.227 \times 10^7 \text{ m}$$

for all these variants. These variants are distinguished by the conditions of passage of the Moon's shadow and pre-shadow at the first two revolutions of the satellite around the earth. The equations of motion of the satellite have been integrated numerically for these three variants. As for the perturbing forces due to the solar radiation pressure we considered four variants:

1. No perturbations due to the solar radiation pressure are presented.
2. No effects due to the Lunar shadow and semi-shadow are taken into account
3. The Lunar shadow has cylindrical form, and
4. The Lunar shadow and semi-shadow effects are taken into account according to the method given above.

In variants 2, 3, and 4, besides the Lunar shadow, also the Earth cylindrical shadow was accounted for, the geopotential was presented by the first term of its development with respect to Legendre polynomials and functions. This term governs the unperturbed motion. Gravitational constant of the Earth has been taken as $3.986013 \times 10^{14} \text{ m}^3/\text{sec}^2$.

Table 1 shows the absolute values of differ-

ences of the position vectors of the satellite (geocentric positions) obtained for the above mentioned variants for the end of the satellite's second revolution around the Earth. The position vector of the satellite is designated as \vec{r}_i , where i is the number of the variant taking into account the solar pressure perturbing forces.

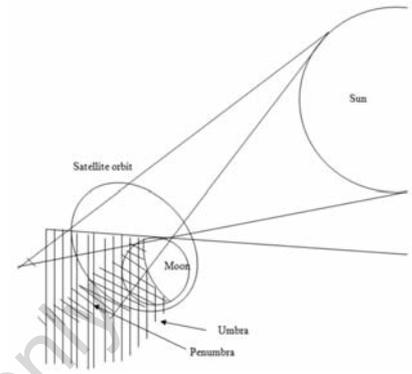


Figure 1. Criteria for penumbra and umbra transits.

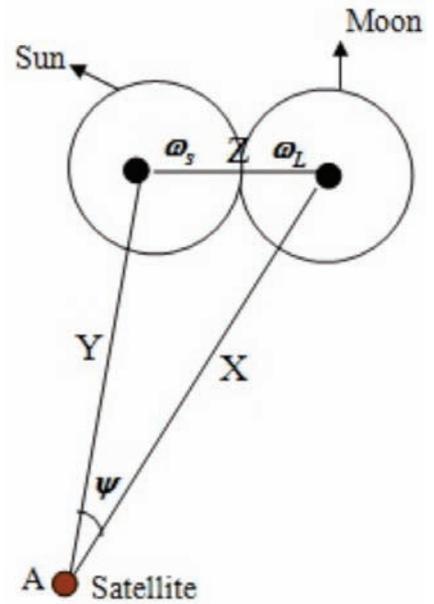


Figure 2. Depiction of the Sun, the Moon and the satellite with the line distances X, Y and Z.

Table 1. Absolute values of differences of the position vectors of the satellite.

Position vector of the satellite	Variants of initial conditions		
	(a)	(b)	(c)
$ \vec{r}_i - \vec{r}_j $	30.1 cm	30.1 cm	30.1 cm
$ \vec{r}_i - \vec{r}_2 $	9.3 cm	4.0 cm	0.9 cm
$ \vec{r}_i - \vec{r}_3 $	4.2 cm	2.6 cm	4.0 cm

Conclusion

The obtained estimation allows us to make the following conclusions;

- i) The influence of the Moon's shadow and semi-shadow must be taken into account when very accurate results are needed.
- ii) The cylindrical lunar shadow can be considered only as a rough approximation to the stricter model that includes the pre-shadow.
- iii) The effect of the cylindrical lunar shadow in the semi-major axes

$$\approx 0.5 \text{ km. for } \frac{A}{m} = 3 \text{ cm}^2 / \text{kg.}$$

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